Bayesian Data Analysis
A PHY 451/551, I CSI 451/551, I INF 451/551
HW2w
Show all work

A Fair 12-sided Die

1. Consider a fair 12-sided die (a dodecahedron) with sides indexed by *i*.

a. What are the possible states of the die? Are these states mutually exclusive and exhaustive?

b. What is the probability of rolling i=10 on the fair 12-sided die?

c. What is the probability of rolling *i* such that *i* is odd?

d. What is the probability of rolling *i* such that *i* is prime?

e. What is the probability of rolling *i* such that *i* is prime and *i* is even?

f. We now introduce a slightly different notation:

π = *i* is prime
e = *i* is even
o = *i* is odd

Note that the probability in part d above can be written as $p(\pi \mid I)$, and the probability in part e above can be written as $p(\pi \wedge e \mid I)$. Since evenness and oddness are mutually exclusive and exhaustive, show that $\pi \wedge (e \vee o) \equiv \pi$. Recall that T is the Truism.

g. Use the sum rule in to write the probability that *i* is prime and *i* is odd, $p(\pi \wedge o \mid I)$, in terms of $p(\pi \mid I)$ and $p(\pi \wedge e \mid I)$. At the end of your derivation, substitute your results from parts d and e to compute the value of this probability.

h. Show that your answer for g is it correct by computing this probability a second way.

2. Independent Pair of Fair 6-Sided Dice
Consider an independent pair of fair 6-sided dice with sides indexed by *i* and *j*.

a. Since they are independent, how does $p(i \mid j, I)$ relate to $p(i \mid I)$?

b. What is the probability of rolling *i*=2 on the first 6-sided die?
That is, what is $p(i = 2 \mid I)$?

c. What quantity does $p(i = 2,\ j = 4 \mid I)$ represent? And what is its value?

d. What is the average value (also called the expected value) of *i* ?

e. Is it possible to ever observe this expected value? Why or why not?

f. What is the expected value of *i*+*j* ?

g. What is the most probable value of *i*+*j* ?

3. Coupled Dice

Imagine now that this pair of 6-sided dice (faces indexed by *i*) is connected via a spring that is attached to the $i = 1$ face and the $j = 1$ face. Since the spring is in the way, we will never be able to roll a 1 or a 6 on either die. Moreover, since the 2 face is opposite to the 5 face and the 3 face is opposite to the 4 face, the spring will have to be twisted quite a bit. The states where opposite faces appear on each die is one half as possible than adjacent faces appearing on each die, which, again due to the twisted spring, is one-half as possible as like faces. That is:

$p(i = 1 \mid I) \;=\; 0$ and $p(i = 6 \mid I) \;=\; 0$

$p(i = 2,\ j = 2 \mid I) \;=\; 2 \times p(i = 2,\ j = 3 \mid I) \;=\; 4 \times p(i = 2,\ j = 5 \mid I)$

a. Write out the probabilities for all the possible cases.

b. Show that the probabilities sum to unity.

c. What is the expected value of $i+j$?

d. What is the probability $p(i \mid I)$ for all values of i ?

e. What is the expected value of i ?