Light Intensity Variations in Planetary Transits

by

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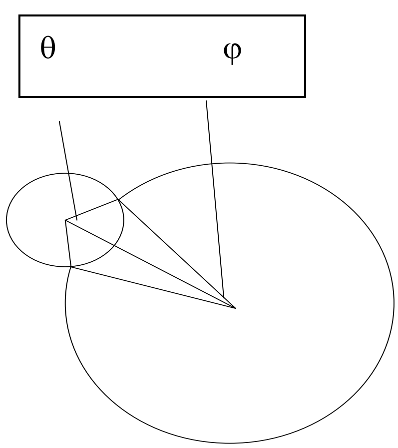
Abstract

The detection of planetary transits in photometric light-curves is poised to become the main method for finding substantial numbers of terrestrial planets. Transit signals from terrestrial planets are very small (ΔF˜10^{-4}), short (T ˜10 hours) dim, which repeat with periodicity of a few months, in time series lasting up to a few years. The reliable and automated detection of such signals in large numbers of light curves affected by different sources of noise is a statistical and computational challenge. We tried an algorithm based on a Bayesian approach. The algorithm is based on the Nested Sampling technique. We use the code of J. Skilling using same technique developed for the detection of light house. This algorithm gave us very good results.

Introduction

The search for rocky, terrestrial planets around other stars is a key research topic in astrophysics for the this and next decade. The most promising approach for the detection of (significant numbers) of terrestrial planets around stars other than the Sun appears to be the search for planetary transits, i.e. dips in the light curve of the parent stars caused by the planet transiting in front of the stellar disk. The light dimming caused by the transit is also small. I used the geometry to find the overlapped area of the planet and star. This is shown in Fig-1.



$$\varphi = \frac{2(R^2 + C^2 - r^2)}{2RC}$$

$$\theta = \frac{2(r^2 + C^2 - R^2)}{2rC}$$

$$OverlappedArea = \frac{1}{2}\left[\varphi R^2 - R^2 Sin(\varphi) + \theta r^2 - r^2 Sin(\theta)\right]$$

Fig-1

Where:
R = Radius of Big Circle
r = Radius of Small Circle
C = Center to Center distance of radii

I developed an algorithm based on the assumption that light dimming depends on the overlapped area during the transit and that simulated the transit of planet and drew the light dimming curve as shown in **Fig2**. This algorithm helped us to write an other algorithm that can predict the dimming of light intensity during the transit. I then use the Nested Sampling algorithm which nicely fitted the data.

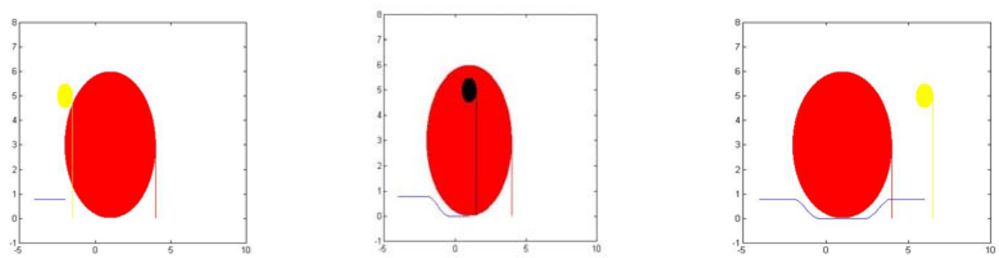


Fig-2

I used the data of Planetary Transit Across Star **HD 209458** Detected by STARE Project Astronomers. This data was downloaded from the website:

http://www.hao.ucar.edu/public/research/stare/hd209458.html

The algorithm
A Bayesian method:

The method employed consists of calculating the likelihood of the data given a certain number of parameters, As we know the error bars in the data so we Assign the Gaussian for the likelihood

$$\Pr ob(X \mid Y, I) = \frac{\Pr ob(Y \mid X, I) \times \Pr ob(X \mid I)}{\Pr ob(Y \mid I)}$$

Considering Evidence = Constant

$$\Pr ob(X \mid Y, I) \propto \Pr ob(Y \mid X, I) \times \Pr ob(X \mid I)$$

where:
$\Pr ob(X \mid Y, I)$ is Posterior Probability

$\Pr ob(Y \mid X, I)$ is Prior Probability
$\Pr ob(X \mid I)$ is Likelihood

For our model we apply the Bayes' theorem as

$$P(Rs, Rp, b, Io, t_i, t_f \mid \{data\}, I) \propto P(Rs, Rp, b, Io, t_i, t_f \mid I) \times P(\{data\} \mid Rs, Rp, b, Io, t_i, t_f, I)$$

Where:
Rs = Radius of star
Rp = Radius of planet
b = Impact parameter
Io = Initial intensity of the start
T_i = Initial time
T_f = Final Time

Now Assigning the Gaussian to the likelihood and Considering the prior constant

$$P(Model \mid I) \approx \prod_{i=1}^{n} \frac{i}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{1}{2\sigma_i^2} \left((\mathrm{Im}_i - F(Rs, Rp, Io, t_i, t_f, t))^2 \right) \right]$$

$$P(Model \mid I) \approx \left(\prod_{i=1}^{n} \frac{i}{\sqrt{2\pi}\sigma_i}\right) \exp\left[-\sum_{i=1}^{n} \frac{1}{2\sigma_i^2} \left((\mathrm{Im}_i - F(Rs, Rp, Io, t_i, t_f, t))^2 \right) \right]$$

Taking log on both sides we will have the log of likelihood as:

$$\log(P(Model \mid I)) \approx \left(\sum \log\left(\frac{i}{\sqrt{2\pi}\sigma_i}\right)\right) - \left[\sum_{i=1}^{n} \frac{1}{2\sigma_i^2} \left((\mathrm{Im}_i - F(Rs, Rp, Io, t_i, t_f, t))^2 \right) \right]$$

Where:
Im_i is the measured value of intensity at some time t
$F(Rs,Rp,Io,t_i,t_f,t)$ is the prediction to the intensity of light at different times.

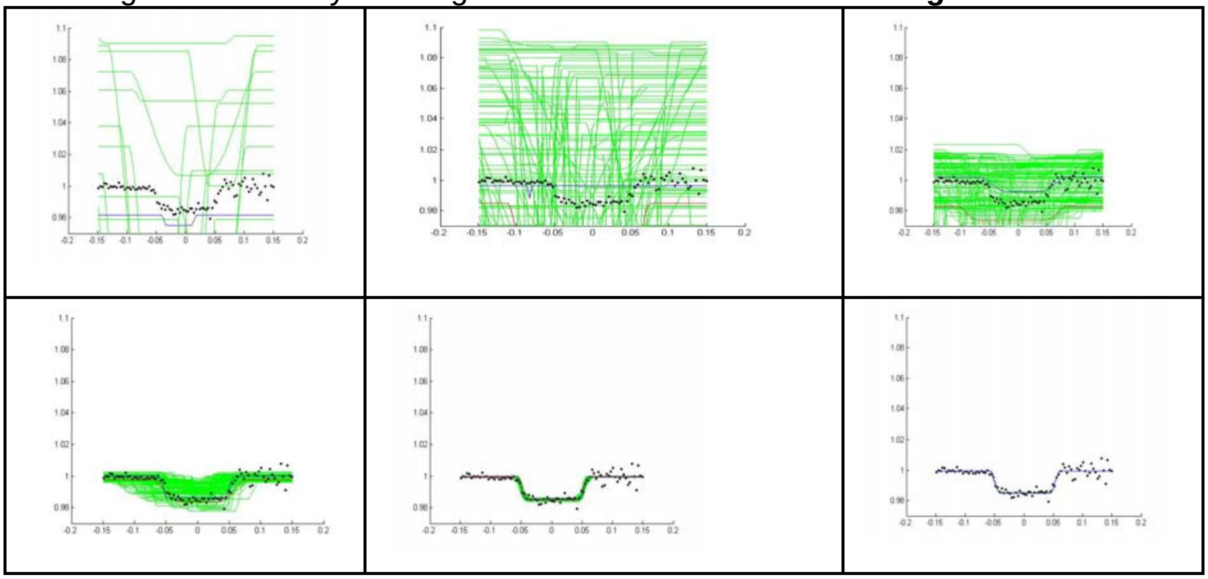
For prediction function we used an algorithm based on the assumption that the light intensity decreases with respect to the overlapped area of planet and star. Also the fact that it decreases with the formula $\Delta I = Io \times (Rp/Rs)^2$.

BY using the Nested Sampling technique in which we used a collection of objects by randomly sampled the prior (based on the existing information), limiting the likelihood to exceed the current value of likelihood.

Assuming the Radius of the Star as **$1.1R_{sun}$** and Radius of planet as **$1.6R_{Jup}$**.This information was used from the work of the STARE Project Astronomers. The initial intensity **Io**, the initial time **t_i** and final time **t_f** is also used from the same data.

I used the J'Skilling C code algorithm that used the same technique to find the position of the light house (Chapter # 9 Data Analysis A Bayesian Tutorial by

D.S.Sivia and J Skilling). By converting this code into Matlab in our Home Work # 5, I have already the algorithm in Matlab. This algorithm sample number of possible solutions then killing the worst one and regenerating the one of the existing then smoothly converges to the solution as shown in **Fig-3**.



Final

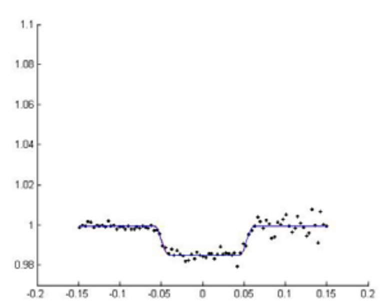


Fig-3

Result

When I plotted the final solution with the original data it fit's the data very well as shown in Fig-4. As it is very much observed from the solution that the intensity seems to be constant at the time when the transit occurred, But it is not the case. To get more accurate results for that period we should have to consider some other aspects like limb darkening.

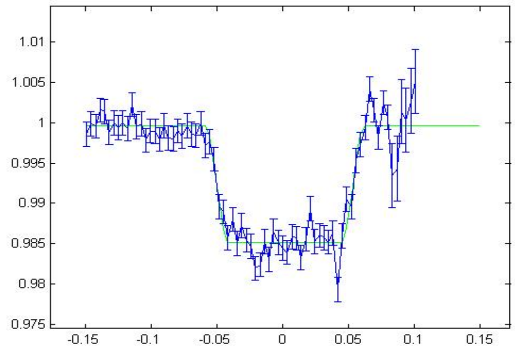


Fig-4

References:

- David Charbonneau, Timothy M Brown, David W. Latham, and Michel Mayor, **"Detection of Planetary Transits Across a Sun-like Star"** Astrophysical J.Lett, 23 Nov 1999.
- **http://www.hao.ucar.edu/public/research/stare/hd209458.html**
- **Data Analysis A Bayesian Tutorial** by D.S.Sivia and J Skilling