If You Are Uncertain About Uncertainty

Most physics students find the concept of uncertainty to be enigmatic; this does not need to be the case. Uncertainty is both an essential and useful component of experimental physics and is not as difficult to master as one may imagine. The following information has been assembled in an effort to help demystify the need for and implementation of uncertainty.

Part of the confusion stems from the fact that in early lab courses a large part of the emphasis is on developing the student’s experimental proficiency. In order to facilitate this goal it is necessary to have a method of measuring the student’s ability to perform an experiment precisely. To this end the student is often provided with an accepted or literature value for the experimental results. The success of the experiment is in part based on whether the student’s experimental results are within a certain range of the accepted value. Although this approach and use of uncertainty is necessary during the development of students and their experimental skills it is not an appropriate approach for actual experiments. In an actual experiment the value determined through experimentation is the correct value. The theoretical value that is being tested may or may not be correct. Determining if the theoretical value is correct is often the purpose of the experiment itself.

In order to understand the appropriate use of uncertainty in Experimental Physics it is necessary to understand the roles played by Theoretical and Experimental Physicists. The theorist working from previous experimental results and accepted concepts looks to expand the body of knowledge by exploring a certain aspect of the physical world. In this process certain assumptions are made, these assumptions are both conceptual and mathematical. As a result of these assumptions the final conclusion reached by the theorist may or may not be correct and needs to be tested.

The experimentalist’s role is to test the theories developed by the theorist. The results provided by the experiment gives the theorist insight into the validity of the theory that has been developed. Although the experimentalist can never prove a theory correct the experimental results can prove a theory wrong. With this role in mind the experimentalist must be honest about the limitations of the equipment and procedures employed during the experiment. These limitations are represented by the uncertainty associated with the final experimental results.

The method employed for a particular experiment is an integral part of the experimental design. The experimenter chooses the most appropriate method of uncertainty analysis based on the conditions and types of limitations that will be encountered during the experiment. Great care needs to be taken during this process to ensure an honest final assessment of uncertainty for the experimental results.

The following information details some of the more common methods of uncertainty analysis. The methods discussed are by no means an exhaustive exploration of uncertainty but is however a sampling of the methods that might be of most use to an undergraduate physics student participating in an advanced lab class.

The easiest uncertainty concept to understand is that of absolute uncertainty. This is the direct uncertainty in a measurement or final value. Absolute uncertainty represents a range of values likely to enclose the true value for a given measurement or result. Imagine a student measuring a table with a meter stick; the meter stick is divided by marks that encompass one millimeter of distance. The
most precise measurement that can be made in this case is plus or minus one millimeter. However if the student utilizes a large caliper with the capability of measuring distances to one micrometer then the uncertainty would be plus or minus one micrometer. Although the concept of measurement uncertainty is the same for each type of measurement made, the exact method to determine that uncertainty differs between types.

There are several means to represent absolute uncertainty. The following are two methods that can be used when writing a paper or report.

measurement ± uncertainty  (Units)     9.812 ± .002 (m/s²)
measurement(uncertainty) (Units)      9.812(2) (m/s²)

The first method is self-explanatory however; in the second method the number in parentheses is the uncertainty in the least significant figure of the measurement. This method is used extensively in tables or lists of literature values.

When adding or subtracting values with the same units the absolute uncertainty of the final value is the absolute uncertainties of the constituent values added in quadrature. This means that the final absolute uncertainty is the square root of the sum of the squares of the constituent absolute uncertainties. This method of adding in quadrature is very familiar to physics students, it is the same method used in the Pythagorean theorem.

\[
\text{If } \quad d = a + b - c \\
\text{Then } \quad \sigma_d = \sqrt{\sigma_a^2 + \sigma_b^2 + \sigma_c^2}
\]

The next type of uncertainty to be considered is relative uncertainty. The relative uncertainty (percent uncertainty) is the ratio of the size of the absolute uncertainty to the size of the measurement value. Imagine the student with the meter stick again; if the student were to measure both the length of a room and a small coin, the one millimeter absolute uncertainty for the meter stick would have a much larger influence on the coins overall uncertainty.

To calculate the relative uncertainty for a measurement, divide the absolute uncertainty by the measurement value and multiply by one hundred.

Relative Uncertainty \( \sigma_{ra} = \frac{\sigma_a}{a} (100) \)

This process can be used in reverse to determine the absolute uncertainty from the relative uncertainty.

Absolute Uncertainty \( \sigma_a = a \frac{\sigma_{ra}}{100} \)

When multiplying or dividing values the relative uncertainty of the final value is the relative uncertainties of the constituent values added in quadrature.
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If \( d = \frac{ab}{c} \)

Then \( \frac{\sigma_{rd}}{100} = \frac{\sigma_d}{d} = \sqrt{\left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2 + \left(\frac{\sigma_c}{c}\right)^2} \)

If the values in question involve exponents then the following form is used.

If \( d = \frac{a^lb^m}{c^n} \)

Then \( \frac{\sigma_{rd}}{100} = \frac{\sigma_d}{d} = \sqrt{\left(l \frac{\sigma_a}{a}\right)^2 + \left(m \frac{\sigma_b}{b}\right)^2 + \left(n \frac{\sigma_c}{c}\right)^2} \)

At times it may be useful to determine the uncertainty in a measurement by repeating the measurement enough times to get a reasonable estimate of the standard deviation. At this point, the uncertainty in any individual measurement is equal to the standard deviation of those measurements. However, if the values are averaged, then the mean measurement value has a much smaller uncertainty, which is equal to the standard error of the mean, which is the standard deviation divided by the square root of the number of measurements, this represents an absolute uncertainty in the average value.

\[ \sigma_{avg} = \frac{\text{stdev}}{\sqrt{n}} \]

Where \( n \) is equal to the number of measurements averaged.

It is hoped that this paper has been successful in alleviating some of the confusion associated with uncertainty analysis. As previously stated it describes only a few of the more common methods of uncertainty analysis available. Although these methods should be sufficient for the average undergraduate lab experiment, the student should always be aware that a more appropriate method might be utilized.