Computational Physics

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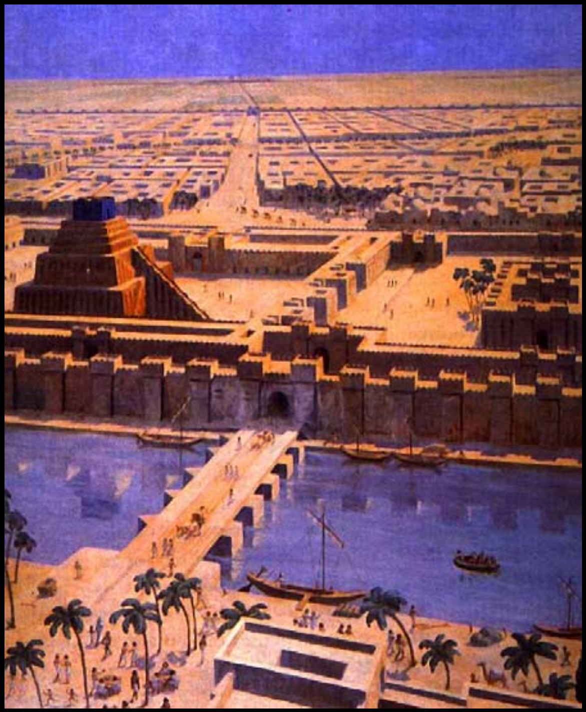
January 2008

Numeric Representation

Excursion to Babylon

Sumer and Babylon



Babylon
(56 mi south of modern Baghdad)

Sumer was the first civilization…home of the first cities.

First written language was probably in Sumer around 3100 BC

In 2500 BC, by Royal Edict, all weights and measures were standardized in Babylon.

Babylonian Measures

Length

1 finger ~ 2/3 inch

1 cubit = 30 fingers

1 cord (surveyors rope) = 120 cubits = 3600 fingers

Weight

1 grain ~ 45 milligrams

1 shekel = 180 grains (¼ ounce)

1 talent = 3600 shekels (67 pounds)

You may note that these numbers are divisible by 6 and 10.

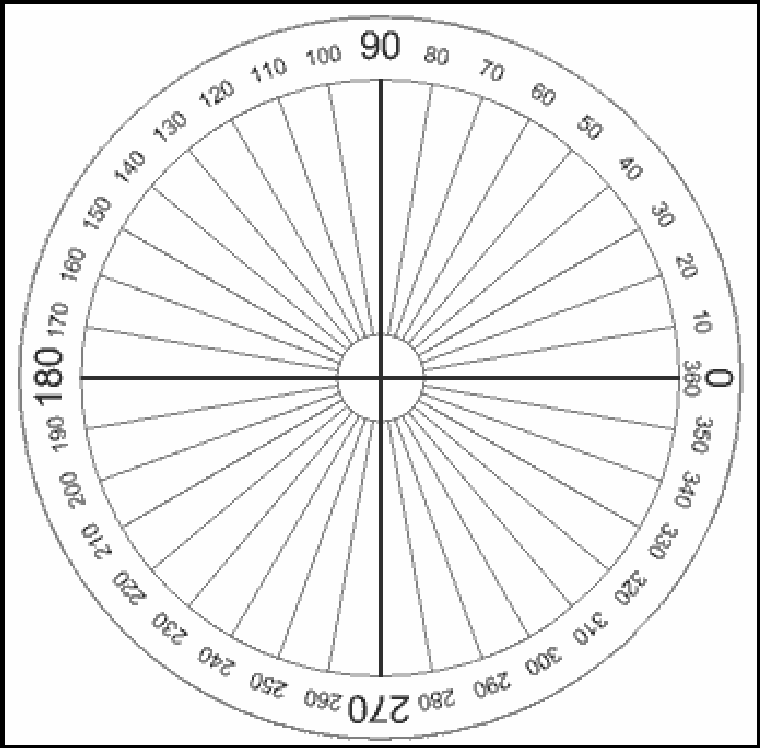
This is a reoccurring theme in the Babylonian system…

Babylonian Measures

Angles

- 1 Circle = 360 Degrees
- 1 Degree = 60 Minutes of Arc
- 1 Minute of Arc = 60 Seconds of Arc

We still use these measures today!



Babylonian Measures

Time

- 1 Day = 24 Hours
- 1 Hour = 60 Minutes
- 1 Minute = 60 Seconds

360 Days per Year

12 Months with 30 Days each

Every 5th year an extra month was added to keep the calendar aligned with the astronomical observations.

Babylonian Measures

Time

- 1 Day = 24 Hours
- 1 Hour = 60 Minutes
- 1 Minute = 60 Seconds

Note that all of these ratios are divisible by 6.

Clearly 6 was important to the Babylonians.

360 Days per Year

12 Months with 30 Days each

Every 5th year an extra month was added to keep the calendar aligned with the astronomical observations.

Our Number System

Look at the numbers 1, 13, 147

The symbol 1 means something different in each number. Its meaning depends on its position in the number.

$1 = 1$

$13 = 1x10 + 3$

$147 = 1x100 + 4x10 + 3 \quad = \quad 1x10^2 + 4x10^1 + 3x10^0$

We call our number system, a Base 10 system since each symbol represents a number between 0 and 9 and is multiplied by the base 10 taken to the power of the place in the expression.

$678{,}197 = 6x10^5 + 7x10^4 + 8x10^3 + 1x10^2 + 9x10^1 + 7x10^0$

Our Number System

John Napier of Merchistoun, Scotland (1550 – 4 April 1617) invented the Decimal Point enabling us to write fractional amounts efficiently.

1/2 becomes 0.5

1/10 becomes 0.1 = 10^{-1}

1/100 becomes 0.01 = 10^{-2}

1/3 becomes 0.33333…
(which goes to show that our system isn't perfect either!)

We often use this to write very large or very small numbers.

The speed of light is 3.0×10^{8} meters/second

Roman Number System

Look at the numbers 1, 13, 147 again

The Romans used different symbols for different quantities.

1 = I

13 = XIII

147 = CIXVII

13 = 10 + 3 = X + III

147 = 100 + 40 + 7 = C + IX + VII

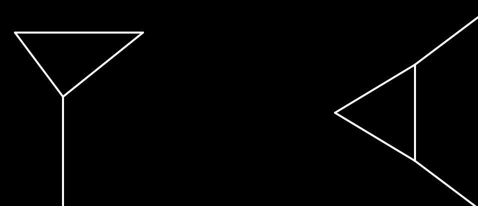
The Romans didn’t have a symbol for 0, which is why they had to use separate symbols for everything.

Babylonian Writing System (Cuneiform)

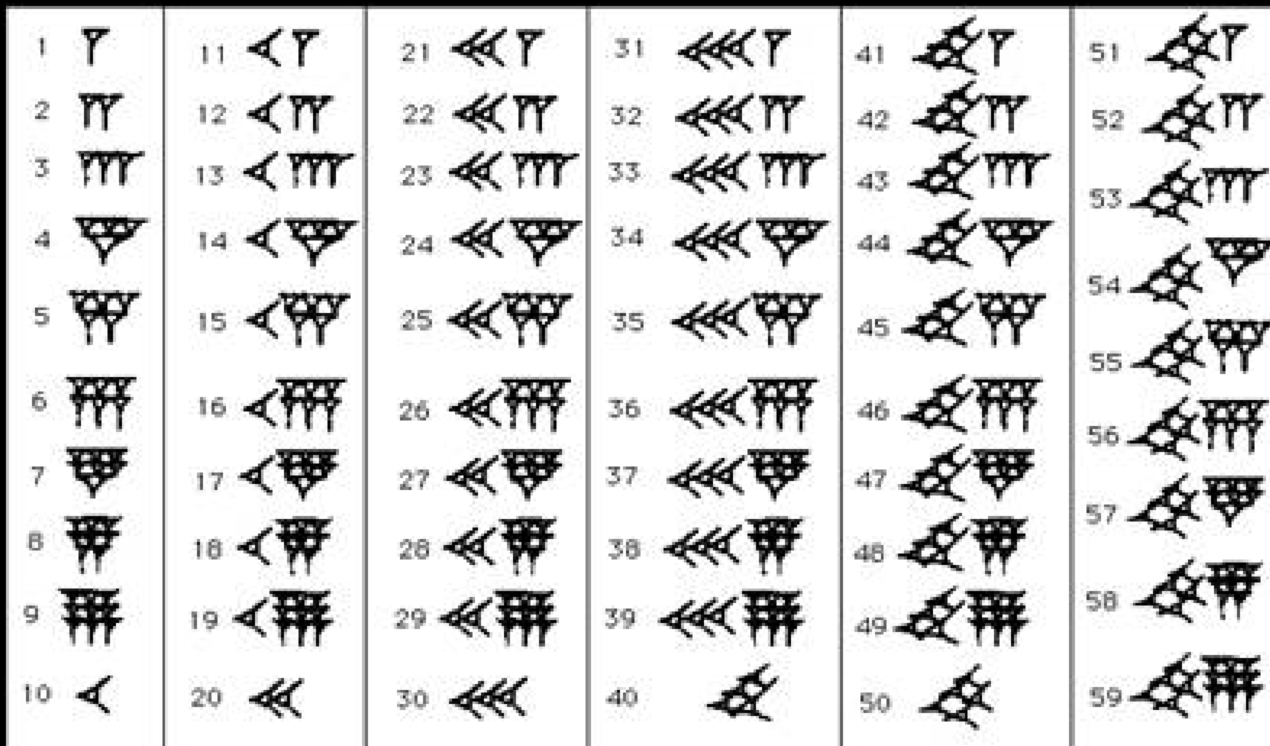
The Babylonians wrote on soft clay tables which were then baked to make the writing permanent.
(don’t make fun…we “burn” DVDs)

▲ Writing on a soft clay tablet with a reed pen.

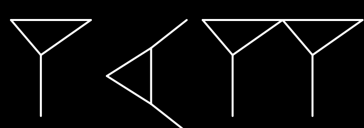
Their written number system (cuneiform) took advantage of this by using stencils with two basic patterns that are repeated to make more complex numbers.



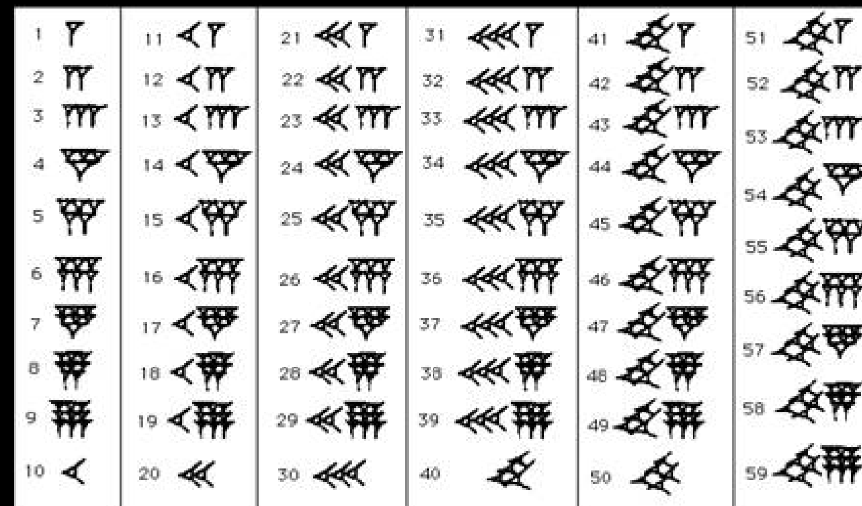
Sexagesimal Number System (Base 60)



At 60, the pattern starts all over again…

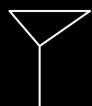
72 =  = 60 + 10 + 2

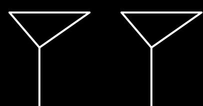
Problems with the Babylonian System

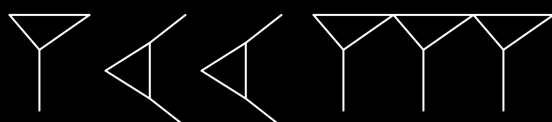


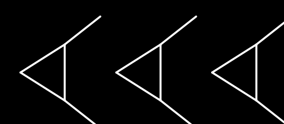
There is no symbol for zero.

So the number 60, is identical to the number 1…

60 = 

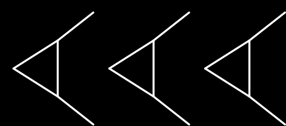
61 = 

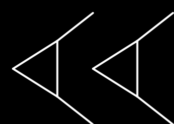
83 = 

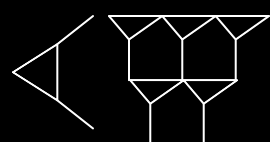
There is no decimal point either, so ½ = 30/60 = 
Indistinguishable from 30

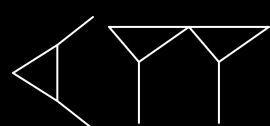
Benefits of the Babylonian System

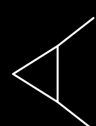
All the simple fractions are easily expressed as a single number

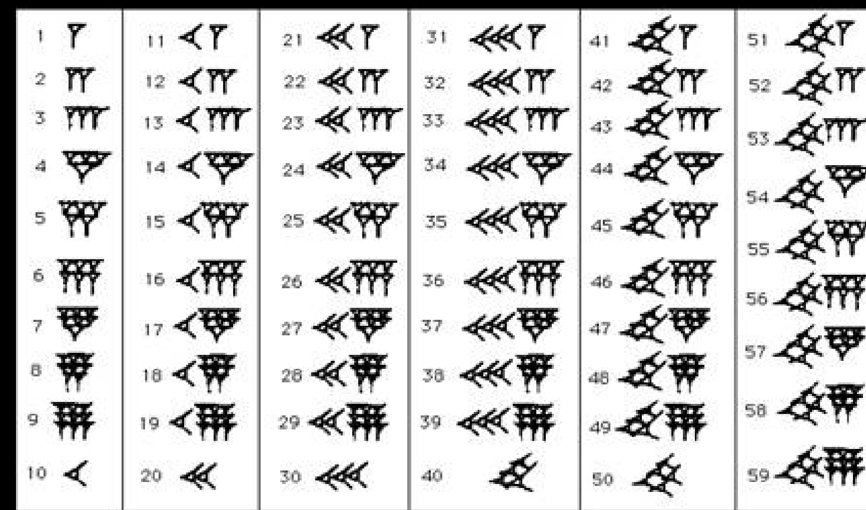
1/2 = 30/60 = 

1/3 = 20/60 = 

1/4 = 15/60 = 

1/5 = 12/60 = 

1/6 = 10/60 = 



Of course, 1/7 is going to cause some problems, but it does in our system too…

1/7 = 0.142857142857…

Metric System

Today's metric system improves on the Babylonian system.
It is base 10, instead of 60.

But we have extra conveniences:
We use 0 to represent zero (nothing)
We use a decimal point to indicate a fractional portion
The name of the unit of measure indicates the scale

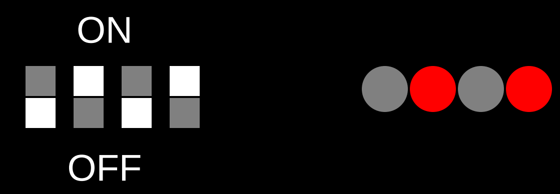
1 meter = 10 decimeters = 100 centimeters = 1000 millimeters
1 kilometer = 1000 meters

Our scientific notation takes advantage of this.

Base 2 Number Systems

Binary switches can be used to store numbers in Base 2 notation:

0101



$0101 = 0x2^3 + 1x2^2 + 0x2^1 + 1x2^0$