

**PHY 151  
Honors**

**FINAL EXAM**

**NAME** \_\_\_\_\_

**DUE: March 13, 2009  
5:00 pm**

**INSTRUCTIONS:** There are 4 questions with multiple parts each valued at 25 points for a total of 100 points.

**You are allowed to use any books or notes, physical or online, at your disposal.**

**You are to do YOUR OWN WORK.**

**Anyone caught cheating or copying another's work will get a ZERO.**

**In all problems, SHOW ALL OF YOUR WORK AND YOUR SOLUTIONS on separate sheets of paper. Just writing down the solution will result in NO CREDIT.**

**Be sure to read through the entire test before beginning. Some parts may be much easier than others, and just because you are finding one part difficult does not mean that you will find the other parts of the question difficult.**

**Additional Note:** Your graded homework solutions are hanging in a folder alongside my office door.

**For the first two problems, you do not need to (and should not) use relativity except where indicated**

1. You are at rest with respect to both an infinite line of charge with charge density  $\lambda$  and a charge  $Q$  located a distance  $r$  from the linear charge density. Choose a coordinate system so that the linear charge density is oriented along the direction of the x-axis.

- a. Find the electric field due to the linear charge density at the position of the charge  $Q$ .
- b. Find the magnetic field at this position.
- c. What is the total force  $F$  on a charge  $Q$  due to the line of charge?

2. You now move at a velocity  $v$  in the direction of the -x-axis. In this frame of reference, the linear charge density is moving in the direction of +x-axis with velocity  $v$  along with the charge  $Q$ .

- a. What is the current due to the relative motion of the linear charge density with respect to your frame of reference?
- b. What is the magnetic field at a distance  $r$  from this current in this new frame of reference?

c. The force on the charge  $Q$  is perpendicular to your direction of motion and in this new reference frame this force is in the same direction as in question 1, but with magnitude equal to  $\gamma F$  where

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$  and  $F$  is the force from question 1. Use this fact to show that the electric field that

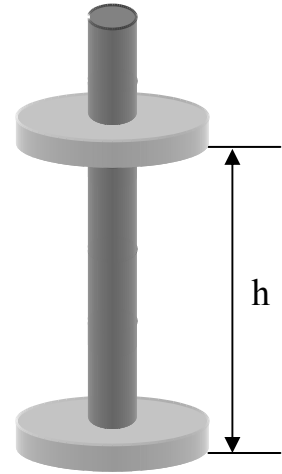
you see in this frame at a distance  $r$  from the linear charge density is given by  $\gamma E$ , where  $E$  is the electric field found in question 1. Note that the component of  $E$  perpendicular to the direction of motion is altered, but the parallel component is not.

Notice that in both frames of reference the charge  $Q$  feels a force  $F$ . In the first frame it is due only to an electric force. In the second frame it is due to both an electric and magnetic force. It is in this sense that the two forces are different aspects of a single electromagnetic force.

3. The magnetic field from a magnetic dipole is given by

$$\mathbf{B}_{dip}(\mathbf{r}) = \frac{\mu_o}{4\pi} \frac{1}{r^3} [3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}]$$

where  $\mathbf{m}$  is the magnetic moment,  $r$  is the distance between the dipole and the point where the field is being evaluated, and  $\hat{\mathbf{r}}$  is a unit vector that points from the center of the dipole to the point where the field is being evaluated. That big dot indicates the vector dot product.



- a. Given that the potential energy of a dipole in a magnetic field is given by  $U = -\mathbf{m} \cdot \mathbf{B}$ , show that the potential energy between two magnetic dipoles  $\mathbf{m}_1$  and  $\mathbf{m}_2$  can be expressed as

$$U = \frac{\mu_o}{4\pi} \frac{1}{r^3} [\mathbf{m}_1 \cdot \mathbf{m}_2 - 3(\mathbf{m}_1 \cdot \hat{\mathbf{r}})(\mathbf{m}_2 \cdot \hat{\mathbf{r}})].$$

- b. A magnetic toy consists of two donut-shaped permanent magnets each with mass  $M$  that slide without friction on a vertical pole. Approximate the magnetic field of each magnet as a dipole both with dipole moments of magnitude  $m$ . If the dipoles are oriented opposite to one another, one magnet will hover above the other. Write the total potential energy of the system by accounting for both the magnetic dipole interaction and the gravitational potential energy of the top magnet as it hovers a height  $h$  above the other. Hint: Take the time to get the vectors drawn properly on the figure and compute the dot products in the expression for the magnetic potential energy above. Remember that  $\hat{\mathbf{r}}$  is a unit vector.

- c. Show that the height of the top magnet is given by  $h = \left( \frac{3\mu_o m^2}{2\pi M g} \right)^{1/4}$

- d. If the mass of each magnet is 25gm and the top magnet hovers at a height of 3cm, what is the magnetic moment  $m$  of the magnets?

4. A small current loop creates a dipolar magnetic field, which we describe by its magnetic moment  $\mathbf{m} = I\mathbf{a}$ , where  $I$  is the current in the loop and  $\mathbf{a}$  is the vector area, which is a vector pointing perpendicular to the loop in the direction dictated by the right-hand rule and with a length proportional to the area of the loop.

- a. What is the magnetic moment of a circular loop of wire with radius 1cm and current 1A?
- b. If instead of a single loop, we loop the coil around 10 times, what is the resulting magnetic moment?
- c. How many loops of wire with radius 1cm and current 1A would one need to replace the permanent magnet in the problem above?