FQXI 2014: Foundations of Probability and Information

CONTINGENT or DERIVABLE

To what degree are the Laws of Physics dictated yet arbitrary (i.e. must be discovered)?
and to what degree are Laws of Physics derivable?

Clearly since Newton, we have come to believe that there exists a small set of fundamental principles from which others can be derived. How small is that set?

ROLE OF THE OBSERVER

To what degree are the Laws of Physics observer-based and precisely what does this mean?

Many quantities in physics are observer-based: distances, times, speeds, energy, momentum, spin.
Some are invariant: mass, charge
Why do so many quantities depend on the observer? Is physics really about observers making optimal predictions? (see next question)

ROLE OF INFORMATION

To what degree do the Laws of Physics reflect consistent/optimal rules for information processing?

Information appears to play a central role in some areas of physics: quantum mechanics, statistical mechanics, black holes
How far does this go? Electromagnetism? Gravity?
And what does that tell us about the role of the observer?

Foundations of Probability and Information

Foundations matter!

A good foundation forms a broad base on which theories can be constructed.

A narrow foundation limits perspective and scope

Probability Theory

Kolmogorov – measures on sets

DeFinetti – consistent betting

Cox – generalization of Boolean logic (degrees of belief)

Ranking Logical Statements – Order Theory (degrees of inclusion)*

Entropy and Information

Boltzmann – combinatorics

Gibbs – thermodynamic or statistical entropy

Shannon – communication channel

Ranking Questions – Order Theory (degrees of inclusion)*

* Knuth & Skilling, 2012. *Axioms* 1(1):38-73, arXiv:1008.4831 [math.PR]

** Knuth 2005. Neurocomputing. 67C: 245-274.

Clues to a Broader Context

An Apparent Pattern (inclusion-exclusion)

$\Pr(A \vee B \mid C) = \Pr(A \mid C) + \Pr(B \mid C) - \Pr(A \wedge B \mid C)$	Probability
$I(A;B) = H(A) + H(B) - H(A,B)$	Mutual Information
$Area(A \cup B) = Area(A) + Area(B) - Area(A \cap B)$	Areas of Sets
$\max(A,B) = A + B - \min(A,B)$	Polya's Min-Max Rule
$\log LCM(A,B) = \log A + \log B - \log GCD(A,B)$	Integral Divisors
$I_3(A,B,C) = \lvert A \sqcup B \sqcup C\rvert - \lvert A \sqcup B\rvert - \lvert A \sqcup C\rvert - \lvert B \sqcup C\rvert + \lvert A\rvert + \lvert B\rvert + \lvert C\rvert$	Amplitudes from three-slits (Sorkin arXiv:\\gr-qc/9401003)

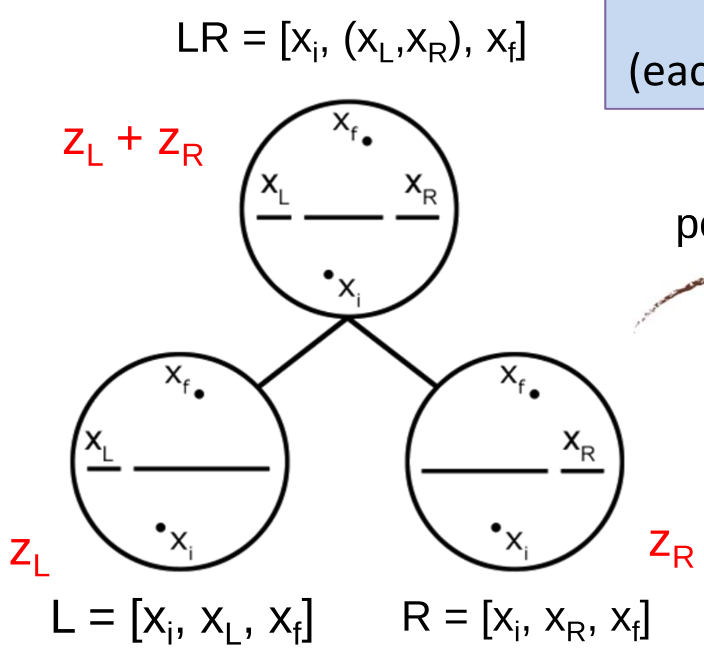
"the essential content of both statistical mechanics and communication theory, of course, **does not lie in the equations; it lies in the ideas that lead to those equations"** E. T. Jaynes, 1958

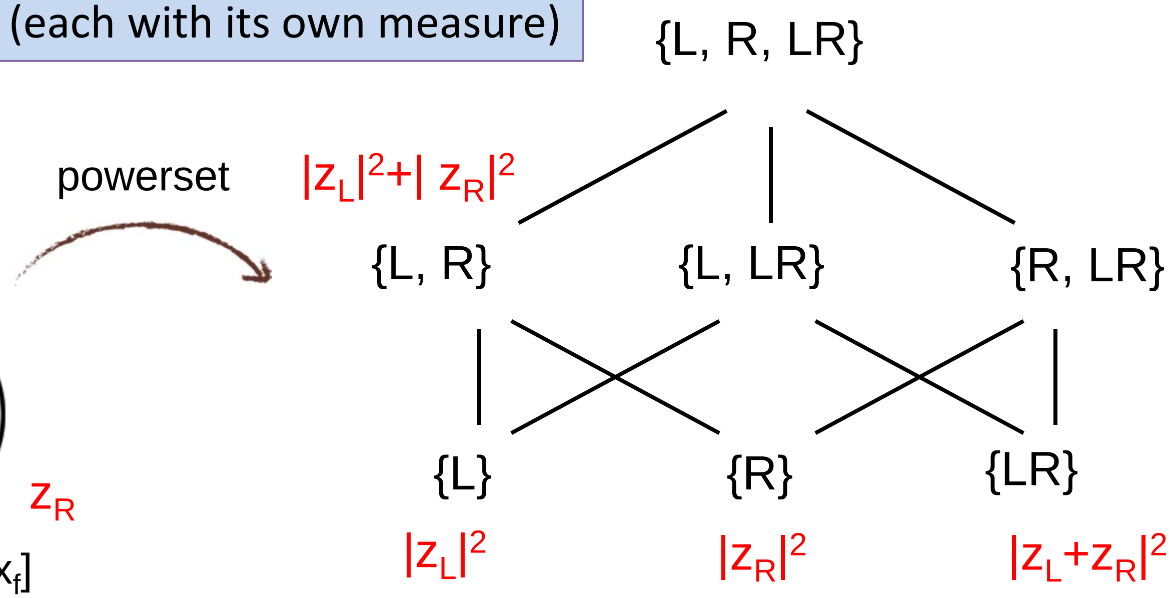
The relations above are constraint equations ensuring consistent quantification in the face of certain symmetries (in this case, associativity along with a notion of ordering).

Knuth, 2003. Deriving Laws, arXiv:physics/0403031 [physics.data-an]
Knuth, 2009. Measuring on Lattices, arXiv:0909.3684 [math.GM]

Quantum Mechanics and Probability

Two Spaces
(each with its own measure)





Measurement Sequences
(quantified by amplitudes)

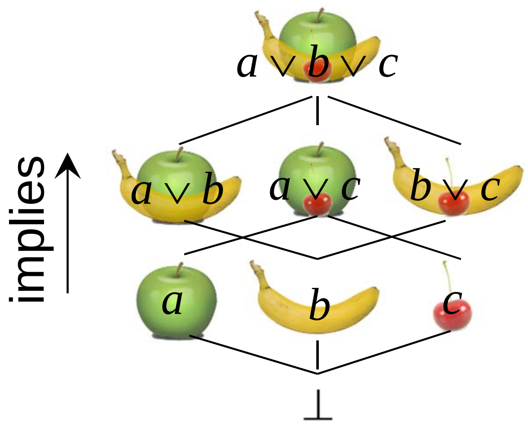
Statements about Measurement Sequences
(quantified by probabilities)

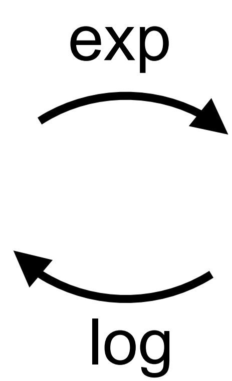
Quantum mechanics is not only consistent with, but relies on probability theory.
Amplitudes encode information about the relationships among measurement sequences to enable consistent assignment of probabilities.

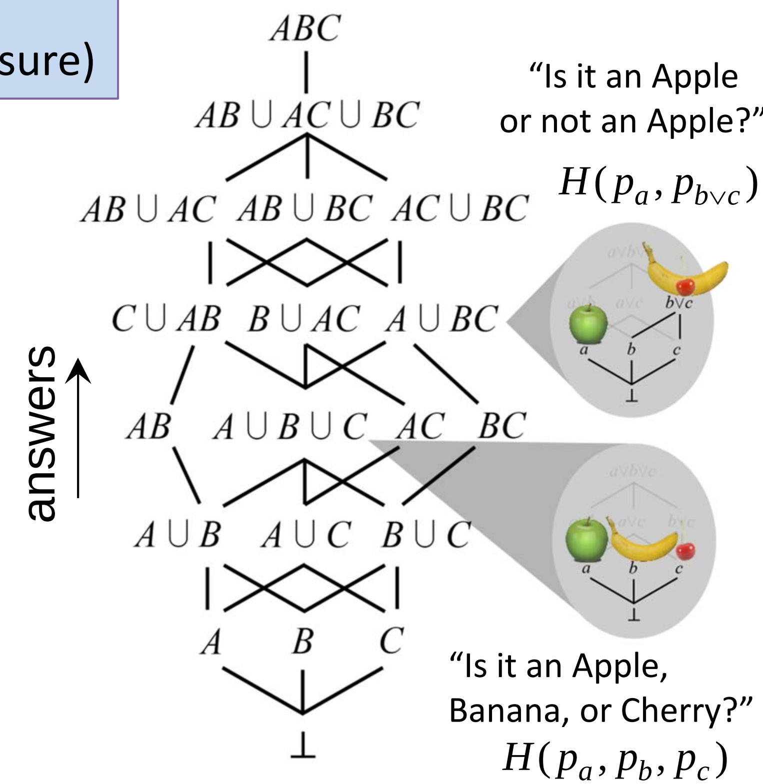
Goyal, Knuth, Skilling. Phys. Rev. A 81, 022109 (2010)
Goyal, Knuth. Symmetry 3(2), 171-206 (2011)

Questions and Information

Two Spaces
(each with its own measure)







Statements (about fruit)
(quantified by probabilities)

Questions (about fruit)
(quantified by entropy and information)

Both spaces (statements and questions) have sum rules, product rules and a Bayes Theorem.
Generalized probabilities are not called for. Again, the spaces work together.

Knuth 2005. Neurocomputing. 67C: 245-274 (A Better Reference is Forthcoming)

Important Points

- Probability Theory deals with inferences among logical statements.
- The Sum and Product Rules of Probability Theory are constraint equations enforcing consistency with symmetries.
- Probability Theory does NOT describe HOW to assign probabilities to atomic statements.

- Quantum mechanics is not only consistent with, but relies on probability theory.
- Amplitudes encode information about the relationships among measurement sequences to enable consistent assignment of probabilities, and as such they enable the consistent assignment of probabilities of statements about measurement sequences.

- The two measures, amplitudes and probabilities, are associated with different spaces:
- The space of measurement sequences and
  The space of statements about measurement sequences.

- Quantum mechanics is not a generalized form of inference… it is simply inference.
- Its not probability theory that is generalized, but rather a theory of measures which enables a wide array of applications, some of which work in concert together.